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T A H A L   W A T E R   P L A N N I N G   F O R   I S R A E L
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GENERATION OF MONTHLY INFLOWS INTO LAKE KINNERET AND YARMOUK RIVER

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PREFACE

This report is concerned with providing four sets of 150 years of correlated artificial monthly inflows into the Yarmouk River and the Lake Kinneret.

The generated data is used as the basis of a simulation study which ascertains the worthiness and priority order of projects designed to increase the Kinneret exploitation , a scheme conceived by the Department of Long-Term Planning, Tahal.

Three uncertainty elements are encompassed by the simulation. They are: (i) the question surrounding the success of artificial rain, (ii) whether the Jordanians will build a dam on the Yarmouk,and (iii) whether Israel will release water from the Lake to the Jordan. The most viable priority order of the projects are selected under an off-on policy for each uncertain event,yielding a total of eight possible futures.

In order to account for an increase in rainfall due to cloud seeding over and above the two sets of inflows pertaining to the Yarmouk River and Lake Kinneret, a third and fourth set of generated monthly inflows (related to Lake Kinneret) were necessary. Two values of the annual mean increase, together with its respective standard deviation were thought sufficient in explaining the phenomenon.

TABLE OF CONTENTS

Page

LIST OF TABLES

APPENDICES

GENERATION OF MONTHLY INFLOWS INTO LAKE KINNERET AND THE YARMOUK RIVER

1. INTRODUCTION

A generation procedure was postulated to reproduce, on the average, important statistical parameters of the historic monthly data of the Kinneret and Yarmouk inflows. The available monthly data were collated, in the case of the Kinneret, from December 1928 to November 1970 (see reference (1) pp. 3-6), as shown in Table 1, and from December 1926 to November 1962 for the Yarmouk, listed in Table 2. The inflows were considered to be a sample from an underlying theoretical probability distribution, and the generation of simulated flows was centered around the production of a random number from a particular distribution, which was transformed into a value of a monthly inflow by means of a linear response function, dependent upon already evaluated simulated flows. The parameters were chosen with a view to preserve the respective monthly means, standard deviations and skewness coefficients, as well as the cross-correlation coefficient (i.e. the correlation between the Kinneret and Yarmouk inflows in a particular year) and the Lag 1 correlations (the correlations between two successive monthly inflows into both the Kinneret and into the Yarmouk). All these estimated parameters are given as part of Tables 8 and 9.

2. SIMULATED KINNERET INFLOWS

The generation of monthly inflows into the Kinneret was based upon a system of equations formulated for TAHAL by Kahan (see reference (1) p. 24). Being autoregressive in structure, the number of lags introduced into the model depended upon the conditional variance of the dependent variable. These equations are given in Table 3. Coefficients of skewness, illustrating the degree of asymmetry of the data, were calculated for each month, the results of which are given in Table 4. Based upon the magnitude of skewness, together with Goodness-of-Fit Tests, it was decided to use one of two probability distributions as the underlying theoretical population from which the historic samples were supposedly drawn, namely the symmetrical normal distribution and the gamma distribution. The latter is asymmetrical, in this case skewed to the right, implying that the values greater than the mean have a larger spread than those which are smaller than the mean.

All simulated inputs were generated by a random variable taken from a normal distribution with zero mean and unit variance. For the months which were considered skewed, namely, December, January, April, November, it was required to transform this random variable into one following the gamma distribution. It was determined in either of two ways, depending on whether its skewness (not that of the corresponding month) was greater or less than 3.0. Appendix 1 provides a means of calculating the skewness of the random variable. If less than 3.0, the Wilson-Hilferty result, which gives an approximate relationship between a normal random variable and a Chi-square variable (and consequently a gamma variable, for the family of gamma distributions includes the Chi-square distribution as a particular case), could be put into effect, as follows:

Given that $\tilde{t}_j$ is a normal random variable for month j with zero mean and unit variance, then:

$$\tilde{t}_{\gamma j} = \frac{2}{\gamma_j}\left\{1 + \frac{\gamma_j \tilde{t}_j}{6} - \frac{\gamma_j^{\,2}}{36}\right\}^3 - \frac{2}{\gamma_j}$$

Where $\tilde{t}_{\gamma j}$ is a gamma random variable with zero mean and unit variance, and γ_j is its coefficient of skewness, see, for example, Matalas((2) p. 938).

However, if skewness is greater than 3.0 the transformation, for small values of $\tilde{t}_j$, will tend to produce values of $\tilde{t}_{\gamma j}$ that are below the theoretical lower bound of $-2/\gamma_j$ of the true gamma variable.*

Kirby (3) has developed a computer-oriented technique based on the Wilson-Hilferty result which preserves the lower bound of the gamma distribution. When confronted with a high coefficient of skewness, it was to these values that we turned.

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* According to the distribution of the gamma function on $(0,1,\gamma_j)$, the theoretical lowest bound is given by $-2/\gamma_j$. For example, given that $\gamma_j = 4$ a value of $\tilde{t}_j$ of $-5/6$ (which will be exceeded in absolute value once out of five times, on the average) would result in $\tilde{t}_{\gamma j}$ having a value of $-2/\gamma_j$. Thus for any $\tilde{t}_j$ less than $-5/6$ the lowest bound of the gamma distribution would be exceeded.

3. SIMULATED YARMOUK INFLOWS

The inflows of the Yarmouk were analysed in much the same way as those of the Kinneret. Here, however, all but three months have high coefficients of skewness (see Table 7, column (1)) and in order for them to be maintained, only an autoregressive model containing not more than one lag could be envisaged.*

However, a problem now presents itself. Although it was deemed sufficient for the sake of the analysis to maintain correlations of only one-lag apart; in order to preserve the annual parameters, and in particular the annual standard deviation, it was necessary to include all lags into the model, providing they are significant, and thus to include the within-year correlation terms. This is because the annual variance is made up of the sum of the monthly variances, together with all the inter-month covariances.

This restriction applies to the months April and May, for in these cases only, the multiple correlation coefficient** becomes significantly larger if another lag is included in the model; however it was thought that in the final analysis the advantages of obtaining a more exact coefficient of skewness outweigh the inclusion of an extra term. Table 5 gives the equations resulting from an autocorrelation model. Using the formula given in Appendix 1, the skewness of the random variables ($t_{\gamma j}$) found in Table 5, have been calculated and are given in column (2) of Table 7.

However, the cross-correlation, i.e. the correlation between the Yarmouk and the Kinneret in a particular month, were not taken into consideration. This was rectified by modifying the autocorrelation equations of the Yarmouk to include the cross-correlations that were found to deviate significantly from zero. By means of Fisher's transformation which is contained in reference (4), a value of r, the sample cross-correlation coefficient, was calculated, as the maximum (within a certain probability error) that the empirical values could take before being considered large enough for the underlying populations to be (in fact) correlated.

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* Appendix 2(b) gives a method of maintaining the coefficient of skewness when a certain inflow is dependent upon two variables. However, because the correlation between the Kinneret and the Yarmouk would have to be taken into consideration, the dependency at this stage is restricted to a one-lag model.

** The multiple correlation coefficient gives the correlation between the dependent variable and the other variables contained in the model. The higher the correlation the better would be the fit.

Fisher showed that if $t = \frac{\sqrt{N-3}}{2} \cdot \frac{(1+r)}{(1-r)}$ (1)

Where N is the sample size (in the case of the Yarmouk N - 34), then t is distributed approximately as Normal distributed on (0,1) under the hypothesis that ρ (the theoretical cross-correlation) = 0.

At the 95% level, that is with a 0.95 probability of accepting the null-hypothesis when correct, the result is significant if $|t| \geq 1.96$. Substituting this value for t in Eq. (1) above, r was found to be significant when $r \geq 0.34$. Thus any positive value of the cross-correlation less than 0.34 was taken as zero.

Appendix 2(a) shows what values the coefficients need to take in order to maintain the appropriate parameters. Inclusion of the cross-correlations affects, however, the coefficient of skewness that needs to be maintained, for it introduces random variables $\tilde{S}_j$ and $\tilde{S}_{\gamma j}$ for some $j = 1, \ldots, 12$. $\tilde{S}_{\gamma j}$ is the random variable that finally produces the flow Y_j and thus its skewness remains to be calculated. The final equations which are used to generate inflows into the Yarmouk river are shown in Table 6.

Appendix 2(b) yields the relationship between the skewness of $\tilde{S}_{\gamma j}$ and $\tilde{t}_{\gamma j}$, a comparison of which is found in col. (2) and col. (3) of Table 7.

Generation of $\tilde{S}_{\gamma j}$ was put into effect by a subroutine illustrated in the computer programme in Appendix 4, which uses linear transformations on Kirby's parameter values in order to maintain the coefficient of skewness of $\tilde{S}_{\gamma j}$ and consequently that of Y_j.

4. KINNERET WITH ARTIFICIAL RAIN FACTORS

As part of the could seeding experiments two more series were generated. Based on inflows into the Kinneret, the two series denoted by Y_{ij} ($i = 1,\ldots 12$; $j = 1,2$) have mean values of 10% and 20% respectively more than the Kinneret inflows given by X_i with annual standard deviations of the increase of 0.051 and 0.056 respectively. These values should be considered only as estimates; for the experiment (at the time of writing) is still in progress.

Because of the inclusion of evaporation, the Kinneret inflows take negative values, particularly for the summer months. In order to increase every inflow by a certain amount, (i.e. both positive and negative flows), the absolute value of X_i must be included in the model. The monthly flows Y_{ij} were then calculated from the equation

$$Y_{ij} = X_i + |X_i|\mu_j + \sigma_j X_i \tilde{t}_i, \text{ where } i = 1,\dots 12 \text{ and } j = 1,2,$$

where μ_j = mean value of the increase in artificial rain, ($\mu_1 = 0.1$, $\mu_2 = 0.2$)

σ_j = monthly standard deviation of this increase, (σ_1, σ_2 to be calculated)

and $\tilde{t}_i$ = independent normal random variable distributed on (0, 1)

It remained to calculate the monthly standard deviation of the increase in both series, for, although the annual deviation is small, the monthly values are known to fluctuate. This can be done, provided that the inflows in every month increase according to the same distribution, that is with a fixed mean (0.1 and 0.2 respectively) and a fixed standard deviation (σ_1, σ_2 respectively), within the year.

The problem is then reduced to solving σ_1, σ_2 in the following equations, which are set up in order to equate the variance of the annual flows with that of the sum of the monthly flows.

$$\text{Var}\{X(1\cdot 1 + 0.051\tilde{t})\} = \text{Var}\{\sum_i (X_i + |X_i| 0.1 + |X_i|\sigma_1\tilde{t}_i)\} \quad (2)$$

$$\text{and} \quad \text{Var}\{X(1\cdot 2 + 0.056\tilde{t})\} = \text{Var}\{\sum_i (X_i + |X_i| 0.2 + |X_i|\sigma_2\tilde{t}_i)\}$$

where $X = \sum_{i=1}^{12} X_i > 0$

and $\tilde{t}$, $\tilde{t}_i$ $i = 1,\dots,12$ are all independent random variables distributed as normal on (0,1).

However, the complications involved in solving the equations seem to outweigh the benefit, for it is possible to approximate them by a simpler set, as follows:

$$\text{Var}\ \{X(1.1 + 0.051\tilde{t})\} = \text{Var}\ \{\sum_i X_i\ (1.1 + \sigma_1 \tilde{t}_i)\}$$

(3)

and $$\text{Var}\ \{X(1.2 + 0.056\tilde{t})\} = \text{Var}\ \{\sum_i X_i\ (1.2 + \sigma_2 t_i)\}$$

It was found (see Appendix 3) that σ_1 and σ_2 can be solved by using the relationships below:

$$\sigma_1 = 0.051 \sqrt{\frac{\text{Var}\ (X) + E^2\ (X)}{\Sigma\ \text{Var}\ (X_i) + \Sigma\ E^2\ (X_i)}} = 0.582$$

$$\sigma_2 = 0.056 \sqrt{\frac{\text{Var}\ (X) + E^2\ (X)}{\Sigma\ \text{Var}\ (X_i) + \Sigma\ E^2\ (X_i)}} = 0.641$$

Thus Y_{ij} of Eq. (2), is generated by using the relationships:

$$Y_{i1} = X_i + |X_i|\ 0.1 + |X_i|\ 0.582\ \tilde{t}_i$$

and $$Y_{i2} = X_i + |X_i|\ 0.2 + |X_i|\ 0.2\ \tilde{t}'_i$$

Where $i = 1,\ldots,12$

$\tilde{t}_i$, $\tilde{t}'_i$ are both standard normal random variables.

5. THE GENERATION PROCESS

The twenty-four derived equations were used as input data for a programme designed to run on an IBM 1130 computer, which is given in Appendix 4. Two subroutines were used. The first - part of the system software - generated random numbers which followed a standard normal probability distribution, whereby a starting value is read into the computer for the process to begin. The second was concerned with maintaining skewness coefficients into the Yarmouk; transforming the normal random variable into a gamma random variable. In this way a sequence of 200 years of synthetic monthly data of Kinneret and Yarmouk inflows were generated. Due to the fact that the inflow in the Kinneret for December was taken as dependent upon the flow in November, an initial value - the mean of November - was used for starting the generation. Consequently the first fifty years of generated results were discarded in the hope that the remaining series would be independent of any starting value.

The magnitudes of the standard deviations of the historic data imply that a long sequence of data is needed for the generated parameters to converge to the historic ("true") values. For the purpose of the main body of the study, it was thought impractical to take more than 150 years of generated data, and due to this constraint, convergence was not automatically effected by the model.

Different initial values needed to generate the random variable were fed into the computer in order to compare the statistical properties of the samples. Fluctuations were produced, as anticipated, in the parameters of the generated sequence. This was particularly evident in the monthly standard deviations and coefficients of skewness.

Because the annual results of the Kinneret and the Yarmouk were considered the more important of the statistical parameters, the generated sequence was chosen to correspond to these values as closely as possible. Tables 8(a), 8(c) and 9 give a comparison between the historic and generated parameters of the Kinneret and the Yarmouk, while Table 8(b) contains the generated means and standard deviations of the two Kinneret series with average artificial rain increases of 0.1 and 0.2.

The first time that generation was carried out, the average of the annual Kinneret values with a 0.1 increase was 630.4, while against this, 1.1 multiplied by the average of the annual basic Kinneret flows gives a value of 621.7. The standard deviation should have been (see Eq. (3.4) in Appendix 3) 263.4 but the generated result was 269.6. Similarly, the increase of flows into the Kinneret by an added factor of 0.2 should have resulted in a mean of 678.2, and in a theoretical standard deviation of 285.3 (found again from Eq. (3.4) in Appendix 3), while the artificial rain series generated a mean of 689.5 and a standard deviation of 292.4. The annual means of the two series were considered the most important parameters to be preserved under generation, and so each inflow of every month was multiplied by 622/630 and 678/690 respectively (when the flow was negative it was divided by these amounts). In this way the annual means were reduced to the theoretical means and the annual standard deviations were also reduced by the same amount.

By this procedure a second set of series was generated, yielding monthly means and standard deviations which are found in Table 8(b).

6. CONCLUSIONS

Tables 8(a) and 8(c) show that the generated monthly and average means and standard deviations of inflows into Lake Kinneret and into the Yarmouk river follow very closely those of the historic data. It should be noted however, that a large monthly historic variance gives rise to a less exact generated sequence, because, in such cases, a period of 150 years is not long enough to assume that the generated values tend towards the "expected values" - the values of the parameters that would be reached had the number of sample outcomes become infinite.

The correlation coefficients between and within the two systems - as shown in Table 9 - converge quite quickly for in only four cases from a total of twenty-five are the generated correlations seen to be significantly different from the historic ones.

The main difficulty, however, in this generation scheme concerns itself with the monthly coefficients of skewness. Previous generated inflows, as noted in Section 5, fluctuated a great deal for different samples of 150 years. December, for example, had a coefficient of skewness that ranged from 5.8 to 1.0, showing that these coefficients are very unstable for such small samples, and have a much slower rate of convergence than the other parameters. Even so, the generated results can still be considered indicative of the values governed by the historic sample.

It should be remembered that the analysis has been carried out with the historic series taken as the "true" sets of values. This however is a fallacy that generation techniques, by necessity, cannot avoid. The historic data of 42 years and 36 years for inflows into Lake Kinneret and Yarmouk river respectively should only be considered a sample from a theoretical infinity of observations and, as such, only reflect approximations of the underlying means, standard deviations, correlation and skewness coefficients. There is no evidence to support a principle inherent in the model that the natural phenomena over the past 50 years (say) will repeat itself - even in the mean. Consequently as long as it can be shown statistically (i.e. with a certain degree of probability) that the generated and historic results could have emanated from the same population the generated values should be considered adequate.

TABLE 1: HISTORIC MONTHLY INFLOWS INTO LAKE KINNERET

(IN MCM)

Year	Dec	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.
1928/29	92.13	143.75	186.64	138.44	102.89	47.53	36.18	22.84	18.01	11.53	11.43	49.10
1929/30	84.12	100.54	136.80	77.16	46.76	29.29	16.80	7.73	5.81	2.50	7.56	21.99
1930/31	69.30	101.57	173.23	117.72	72.70	39.57	25.70	13.88	10.77	5.64	9.00	20.03
1931/32	56.75	47.02	122.68	70.05	47.04	27.87	13.49	3.52	0.93	-1.27	7.56	19.36
1932/33	26.75	42.86	25.35	20.69	12.21	-3.71	-11.24	-15.66	-22.52	-15.58	6.33	0.80
1933/34	16.93	73.71	131.91	75.27	47.73	25.19	7.54	-2.87	-7.62	-1.62	1.79	8.69
1934/35	106.72	109.61	152.90	106.10	101.43	47.40	33.16	18.25	13.07	7.23	11.94	32.78
1935/36	52.53	82.23	103.03	84.15	57.35	33.92	19.73	10.18	7.69	4.24	8.17	58.20
1936/37	97.70	130.27	96.66	78.36	61.41	34.77	21.61	11.32	8.40	4.13	9.82	24.24
1937/38	40.98	126.11	149.55	143.02	70.50	46.28	30.75	17.97	13.74	8.38	10.15	56.38
1938/39	61.64	95.63	126.88	125.27	69.46	37.98	24.58	13.17	9.67	4.89	8.75	29.58
1939/40	67.24	141.61	111.94	101.94	65.91	36.76	24.16	13.46	10.50	5.75	10.68	30.40
1940/41	68.79	98.00	128.02	140.32	65.02	35.96	22.89	12.18	9.53	5.00	8.75	18.98
1941/42	76.16	106.70	99.58	115.57	66.30	37.02	24.30	13.32	10.50	5.86	13.89	47.68
1942/43	48.00	130.86	103.71	168.35	168.11	47.39	34.74	21.18	16.48	10.12	11.02	23.35
1943/44	49.53	135.58	102.48	108.44	62.45	36.67	23.88	13.74	11.18	6.49	9.25	102.85
1944/45	111.92	119.29	149.82	103.73	77.98	39.98	28.86	18.25	15.18	9.71	11.63	32.70
1945/46	60.97	56.97	129.19	88.51	52.93	37.89	24.30	13.03	9.53	4.78	8.54	17.93
1946/47	46.18	117.07	84.99	68.37	43.57	25.19	11.33	2.78	0.46	-1.27	5.31	18.69
1947/48	31.76	26.42	155.77	159.13	70.96	34.49	17.69	5.12	1.56	-1.27	5.24	19.36
1948/49	69.93	123.28	144.30	168.16	193.48	49.86	35.79	20.48	14.79	8.69	12.28	17.28
1949/50	67.08	147.13	108.98	95.07	65.20	47.41	21.56	8.38	3.69	7.03	6.78	29.11
1950/51	22.10	34.23	45.96	51.97	37.04	5.26	4.44	-6.94	-10.32	-2.26	6.83	18.04
1951/52	145.65	83.89	169.66	158.59	60.20	33.57	24.38	11.57	5.40	-6.09	-0.05	10.49
1952/53	36.01	91.86	116.06	175.98	118.80	44.03	36.50	12.84	9.20	5.71	17.38	60.20
1953/54	92.79	182.36	266.97	110.29	21.66	58.10	36.63	17.33	19.75	16.86	19.26	46.16
1954/55	74.34	40.58	56.37	50.58	40.35	22.63	4.86	-0.81	-9.78	-4.05	-6.43	25.41
1955/56	105.79	140.82	91.07	100.11	55.15	36.90	20.61	19.28	10.66	-3.35	11.69	16.92
1956/57	45.16	51.04	104.70	126.54	55.71	44.12	17.87	4.60	1.88	-0.41	5.46	21.10
1957/58	93.81	127.93	99.89	55.35	35.16	19.40	3.33	1.83	6.11	0.04	6.42	7.21
1958/59	39.18	50.31	73.49	90.86	42.68	26.53	6.46	4.25	0.46	1.93	2.54	16.58
1959/60	9.12	72.73	32.48	42.66	31.50	12.40	-4.72	-14.74	-20.10	-17.77	-9.16	12.68
1960/61	14.25	24.16	101.44	37.04	38.57	15.24	-7.16	-11.12	-22.16	-16.39	-3.96	14.76
1961/62	108.83	96.45	108.76	52.76	27.34	14.60	-2.59	-10.68	-8.90	-7.53	4.62	4.73
1962/63	51.96	94.73	122.58	95.26	63.31	58.57	15.62	0.17	-2.26	-1.19	14.10	24.19
1963/64	34.36	28.60	176.45	167.14	77.87	43.25	8.90	2.33	1.81	0.29	5.59	67.02
1964/65	57.04	134.27	126.45	69.90	67.80	28.70	5.25	-0.08	-4.98	0.35	11.44	15.63
1965/66	31.71	78.98	89.44	64.35	38.35	6.08	1.14	-15.42	-10.86	-7.43	11.72	7.12
1966/67	58.61	24.19	133.79	203.11	106.76	59.12	30.73	9.23	3.85	7.75	15.12	31.18
1967/68	61.79	225.64	146.36	91.82	62.69	36.51	12.27	-2.86	-5.58	8.22	5.43	34.07
1968/69	93.23	443.82	192.91	208.25	115.99	71.47	41.55	14.00	14.61	25.14	29.03	36.42
1969/70	42.35	122.04	58.97	169.75	70.32	34.27	14.46	-2.49	-4.09	3.76	5.45	34.07

TABLE 2: HISTORIC INPUTS INTO THE YARMOUK

(MCM)

Year	Dec	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.
1926/27	33	48	230	85	43	13	22	26	19	18	23	19
1927/28	14	15	91	28	12	13	7.5	13	16	19	20	18
1928/29	18	58	330	110	110	100	50	24	14	18	27	28
1929/30	21	30	83	41	26	27	28	25	25	24	25	30
1930/31	40	88	210	73	35	30	23	21	20	16	16	16
1931/32	22	20	130	27	23	24	23	24	24	22	20	20
1932/33	22	22	43	20	17	17	13	17	18	22	19	20
1933/34	21	22	67	28	25	25	25	25	25	26	23	22
1934/35	39	62	250	36	40	25	20	24	24	21	23	25
1935/36	28	26	31	24	21	21	20	21	22	22	24	32
1936/37	55	130	90	30	26	22	21	20	20	20	25	32
1937/38	24	69	160	78	27	26	22	21	21	20	22	40
1938/39	31	50	84	96	42	19	18	18	19	20	18	21
1939/40	32	160	62	45	20	17	16	18	19	19	22	24
1940/41	33	94	76	84	25	20	18	19	18	18	19	21
1941/42	33	110	82	110	27	20	17	17	18	18	19	22
1942/43	21	96	90	110	89	28	22	22	22	27	39	27
1943/44	24	120	59	34	26	23	20	22	26	25	26	39
1944/45	69	240	160	68	29	21	20	19	21	21	24	23
1945/46	25	25	130	46	20	19	13	11	14	16	19	19
1946/47	23	80	78	24	14	14	12	11	12	12	14	20
1947/48	24	27	110	100	24	20	18	18	18	17	18	19
1948/49	33	60	86	81	81	20	19	19	18	18	19	19
1949/50	35	77	63	58	37	23	20	21	20	19	20	21
1950/51	23	26	36	23	20	18	8	18	17	17	27	23
1951/52	150	100	200	160	25	19	18	18	18	22	23	24
1952/53	26	58	100	220	76	16	15	15	16	17	23	27
1953/54	34	160	220	50	46	22	19	20	21	22	25	25
1954/55	29	29	24	30	23	20	20	21	22	21	22	30
1955/56	63	100	57	43	28	26	22	22	22	22	23	22
1956/57	25	40	68	59	24	19	19	20	21	20	22	22
1957/58	36	110	33	24	22	23	21	19	20	21	23	21
1958/59	23	27	44	42	23	20	19	19	20	21	22	21
1959/60	22	27	22	23	21	16	15	18	18	19	19	22
1960/61	23	28	53	24	21	27	18	18	18	18	22	23.5
1961/62	78.8	52.7	74.4	27.8	18.0	18.6	18.2	17.8	18.1	19.8	20.7	16.3

TABLE 3: EQUATIONS USED FOR THE GENERATION OF INFLOWS INTO THE KINNERET

December	$X_1 = 48.769 + 0.572X_{12} + 34.287\tilde{U}_{\gamma 1}$
January	$X_2 = 23.961 + 1.275X_1 + 51.073\tilde{U}_{\gamma 2}$
February	$X_3 = 78.033 + 0.109X_2 + 0.468X_1 + 39.314\tilde{U}_3$
March	$X_4 = 39.431 + 0.444X_3 + 0.131X_2 + 38.805\tilde{U}_4$
April	$X_5 = 8.685 + 0.564X_4 + 24.044\tilde{U}_5$
May	$X_6 = -4.023 + 0.082X_5 + 0.135X_4 + 0.097X_3 + 0.068X_2 + 7.415\tilde{U}_6$
June	$X_7 = -12.255 + 0.723X_6 + 0.074X_5 + 2.947\tilde{U}_7$
July	$X_8 = -7.921 + 0.796X_7 + 2.464\tilde{U}_8$
August	$X_9 = -3.198 + 0.998X_8 + 2.072\tilde{U}_9$
September	$X_{10} = -8.422 + 1.642X_9 + 1.264X_8 - 0.325X_7 + 0.556X_1 + 1.465\tilde{U}_{10}$
October	$X_{11} = 6.371 + 0.596X_{10} + 0.083X_9 + 3.959\tilde{U}_{11}$
November	$X_{12} = 24.489 + 1.195X_{11} + 16.982\tilde{U}_{\gamma,12}$

Where X_i = Flow into the Kinneret in month i

and $\tilde{U}_{\gamma i}$ = Random variable of month i from a gamma distribution with zero mean and unit variance

and $\tilde{U}_i$ = Normal random variable of month i with zero mean and unit variance

TABLE 4: COEFFICIENTS OF SKEWNESS FOR EACH MONTH OF INFLOWS INTO THE KINNERET

Month	Coefficient of skewness of inflow
December	1.24
January	2.85
February	0.49
March	0.36
April	1.60
May	0.27
June	0.30
July	0.54
August	0.77
September	0.24
October	0.07
November	1.63

TABLE 5: INPUTS INTO THE YARMOUK BASED UPON AUTOCORRELATION

$$Y_1 = 34.799 + 24.316\,\tilde{t}_{\gamma 1}$$

$$Y_2 = 40.332 + 0.826Y_1 + 46.315\,\tilde{t}_{\gamma 2}$$

$$Y_3 = 103.511 + 72.264\,\tilde{t}_{\gamma 3}$$

$$Y4 = 35.089 + 0.236Y_3 + 40.479\,\tilde{t}_{\gamma 4}$$

$$Y_5 = 14.949 + 0.302Y_4 + 17.877\,\tilde{t}_{\gamma 5}$$

$$Y_6 = 10.295 + 0.389Y_5 + 11.025\,\tilde{t}_{\gamma 6}$$

$$Y_7 = 9.595 + 0.426Y_6 + 3.275\,\tilde{t}_{\gamma 7}$$

$$Y_8 = 9.819 + 0.781Y_7 - 0.238Y_6 + 2.142\tilde{t}_8$$

$$Y_9 = 6.605 + 0.664Y_8 + 2.135\,\tilde{t}_1$$

$$Y_{10} = 5.131 + 0.757Y_9 + 1.775\,\tilde{t}_{10}$$

$$Y_{11} = 4.342 + 0.891Y_{10} + 3.224\,\tilde{t}_{\gamma 11}$$

$$Y_{12} = 9.296 + 0.652Y_{11} + 4.955\,\tilde{t}_{\gamma 12}$$

Where:

Y_i = Flow into Yarmouk in month i

$\tilde{t}_{\gamma i}$ = Random variable of month i from a gamma distribution with zero mean and unit variance

$\tilde{t}_i$ = Normal random variable of month i distributed with zero mean and unit variance

TABLE 6: EQUATIONS USED FOR THE GENERATION OF INFLOWS INTO YARMOUK

December	$Y_1 = 2.831 + 0.493X_1 + 16.566\tilde{S}_{\gamma 1}$
January	$Y_2 = -13.788 + 0.825Y_1 + 0.507X_2 + 29.413\tilde{S}_{\gamma 2}$
February	$Y_3 = -46.189 + 1.247X_3 + 44.949\tilde{S}_{\gamma 3}$
March	$Y_4 = -28.084 + 0.235Y_3 + 0.592X_4 + 28.972\tilde{S}_{\gamma 4}$
April	$Y_5 = -6.452 + 0.302Y_4 + 0.311X_5 + 13.735\tilde{S}_{\gamma 5}$
May	$Y_6 = 21.181 + 0.388Y_5 + 0.312X_6 + 9.762\tilde{S}_{\gamma 6}$
June	$Y_7 = 9.596 + 0.425Y_6 + 3.275\tilde{S}_{\gamma 7}$
July	$Y_8 = 11.059 + 0.780Y_7 - 0.237Y_6 - 0.192X_8 + 0.453\tilde{S}_8$
August	$Y_9 = 6.605 + 0.664Y_8 + 2.135\tilde{S}_9$
September	$Y_{10} = 5.129 + 0.757Y_9 + 1.775\tilde{S}_{10}$
October	$Y_{11} = 4.345 + 0.890Y_{10} + 3.224\tilde{S}_{\gamma 11}$
November	$Y_{12} = 3.496 + 0.652Y_{11} + 0.205X_{12} + 2.755\tilde{S}_{\gamma 12}$

Where X_i = flow in month i into the Kinneret

and where Y_i = flow in month i into the Yarmouk

and $\tilde{S}_{\gamma i}$ = random variable of month i from a gamma distribution with zero mean and unit variance

and $\tilde{S}_i$ = normal random variable of month i distribution with zero mean and unit variance

TABLE 7: COEFFICIENTS OF SKEWNESS ASSOCIATED WITH THE YARMOUK

Month \ Variables*	Y_i (1)	$\tilde{t}_{\gamma i}$ (2)	$\tilde{S}_{\gamma i}$ (3)
December	3.31	3.31	8.94
January	1.38	1.51	0.89
February	1.36	1.36	5.65
March	1.82	2.23	5.82
April	2.15	3.41	6.34
May	4.95** (3.4)	9.13 (5.95)	12.60 (8.21)
June	2.41	-8.36 (1.02)	-8.36 (1.02)
July	-0.47	0.0	0.0
August	-0.07	0.0	0.0
September	0.09	0.0	0.0
October	1.69	3.70	3.70
November	1.32	1.69	4.27

* Y_i is inflow into Yarmouk in month i; for $\tilde{t}_{\gamma i}$, $\tilde{S}_{\gamma i}$ see Tables 5 and 6.

** The coefficient of skewness of May was found to be so high (4.95) as to produce large negative coefficients in June, for the two months are interrelated. Because of difficulties in maintaining negative coefficients, a value of 3.4 was introduced (instead of 4.95) as the maximum that could be utilized in practice; this resulted in the values stated in brackets.

TABLE 8(a): COMPARISON BETWEEN THE HISTORIC AND GENERATED STATISTICAL PARAMETERS OF THE KINNERET

(MCM)

Month	K I N N E R E T					
	M e a n		Standard deviation		Skewness coeff.	
	Historic	Generated	Historic	Generated	Historic	Generated
Dec.	64.8	63.7	36.1	35.8	1.2	0.8
Jan.	106.5	106.1	68.7	59.4	2.8	1.0
Feb.	120.0	122.4	45.3	44.0	0.5	-0.1
Mar.	106.6	107.6	46.3	50.3	0.3	-0.1
Apr.	68.8	66.9	35.5	31.6	1.6	0.2
May	34.9	34.5	15.3	15.8	0.3	0.1
June	18.0	17.5	13.1	13.4	0.3	0.0
July	6.4	6.2	10.6	11.0	0.5	-0.1
Aug.	3.2	3.1	10.7	11.2	0.8	-0.1
Sep.	2.2	2.3	8.2	9.2	0.2	0.0
Oct.	8.0	7.5	6.9	7.5	0.0	0.0
Nov.	28.1	27.2	19.6	17.7	1.6	0.8
Annual	567.6	565.2	237.7	235.9	0.6	0.7

TABLE 8(b): THE GENERATED STATISTICAL PARAMETERS OF THE KINNERET WITH ARTIFICIAL RAIN FACTORS

(MCM)

Month	10% increase in Kinneret inflows*				20% increase in Kinneret inflows**			
	Mean		Standard deviation		Mean		Standard deviation	
	Generated	Theoretical	Generated	Theoretical	Generated	Theoretical	Generated	Theoretical
Dec.	70.2		55.8		76.2		60.9	
Jan.	119.4		86.7		129.7		94.7	
Feb.	140.7		92.3		152.9		100.8	
Mar.	115.2		84.9		125.1		92.6	
Apr.	69.4		57.5		75.3		62.8	
May	36.3		26.4		39.3		28.9	
June	20.2		20.1		22.0		21.7	
July	8.1		12.7		9.6		13.4	
Aug.	3.1		12.0		4.0		12.4	
Sep.	2.7		9.7		3.4		10.1	
Oct.	7.3		9.3		8.0		10.0	
Nov.	29.6		25.6		32.2		29.6	
Annual	622.1	621.7	266.4	263.4	677.3	678.2	287.6	285.3

* The factor 1.1 denoting the increase in flows has a standard deviation of 0.583 per month

** The factor 1.2 denoting the increase in flows has a standard deviation of 0.641 per month

TABLE 8(c): COMPARISON BETWEEN THE HISTORIC AND GENERATED STATISTICAL PARAMETERS OF THE YARMOUK

(MCM)

Month	YARMOUK					
	Mean		Standard deviation		Skewness coeff.	
	Historic	Generated	Historic	Generated	Historic	Generated
Dec.	34.8	34.6	24.3	23.3	3.3	1.6
Jan.	69.1	70.8	49.9	52.7	1.4	0.7
Feb.	103.5	106.4	72.3	65.3	1.4	0.5
Mar.	59.5	58.5	43.4	46.8	1.8	0.6
Apr.	32.9	31.6	22.0	25.1	2.1	0.8
May	23.1	22.6	13.8	11.3	4.9	3.2
June	19.4	19.8	6.7	6.1	2.4	2.0
July	19.5	20.0	3.5	3.5	-0.5	0.4
Aug.	19.6	20.0	3.1	3.2	-0.1	0.2
Sep.	19.9	20.1	3.0	3.1	0.1	-0.2
Oct.	22.1	22.0	4.1	3.6	1.7	0.4
Nov.	23.7	23.2	5.6	4.8	1.3	0.9
Annual	447.2	449.5	155.7	162.0	0.9	0.2

TABLE 9: COMPARISON OF HISTORIC AND GENERATED CORRELATION COEFFICIENTS BETWEEN THE MONTHLY INFLOWS OF KINNERET AND YARMOUK

Month		Dec.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.
Corr (X_i, Y_i)	historic	0.73	0.70	0.78	0.81	0.82	0.26	0.32	0.22	0.11	0.06	0.24	0.72
	generated	0.79	0.69	0.81	0.82	0.81	0.12	0.15	-0.53	-0.30	-0.22	-0.10	0.73
Corr (Y_i, Y_{i-1})	historic	0.22	0.40	0.22	0.39	0.60	0.62	0.88	0.67	0.74	0.81	0.64	0.48
	generated	not calculated	0.70	0.25	0.70	0.83	0.63	0.82	0.65	0.73	0.81	0.62	0.50
Corr (X_i, X_{i-1})	historic	0.31	0.67	0.41	0.51	0.74	0.72	0.86	0.97	0.98	0.86	0.82	0.37
	generated	0.31	0.67	0.40	0.50	0.74	0.64	0.96	0.97	0.98	0.92	0.84	0.44

Note: X_i = Flow into Kinneret in month i

Y_i = Flow into Yarmouk in month i

Thus Corr (X_i, Y_i) = Correlation between Kinneret and Yarmouk in month i

Corr (Y_i, Y_{i-1}) = Correlation in Yarmouk between month i and preceding month

Corr (X_i, X_{i-1}) = Correlation in Kinneret between month i and preceding month

APPENDIX 1

CALCULATION OF THE COEFFICIENT OF SKEWNESS OF THE RANDOM VARIABLE IN A SINGLE REGRESSION MODEL

An autoregressive Lag 1 equation which will preserve, after large-sample generation, the means, standard deviations and autocorrelation coefficient of an inflow X_j, in month j, is given by:

$$X_j = \mu_j + \frac{\sigma_j}{\sigma_{j-1}} \rho_X (X_{j-1} - \mu_{j-1}) + \tilde{t}_j (1-\rho_X^2)^{\frac{1}{2}} \sigma_j \qquad (1.1)$$

where μ_j is the mean of X_j, σ_j is its standard deviation, ρ_X is the Lag 1 correlation coefficient, $\tilde{t}_j$ is a random variable distributed as N (0,1).

Let X_j be an input following a skewed (gamma) distribution, with coefficient of skewness $\gamma(X_j)$,

$$\text{where } \gamma(X_j) = \frac{E\{(X_j-\mu_j)^3\}}{[E(X_j-\mu_j)^2]^{3/2}}$$

The skewness can be maintained by providing for the generation of $\tilde{t}_j$ to be independent of X_j, and from a standard gamma distribution with skewness coefficient $\gamma(\tilde{t}_j)$. $\gamma(X_j)$ is found empirically from the data, from which $\gamma(\tilde{t}_j)$ is to be estimated.

Making a transformation in Eq. (1.1):

$$Z_j = \frac{(X_j - \mu_j)}{\sigma_j}$$

in order for $\gamma(X_j) = E(Z_j^3)$ to hold.

Equation (1.1) becomes

$$Z_j = \rho_X Z_{j-1} + \tilde{t}_j (1-\rho_X^2)^{\frac{1}{2}} \qquad (1.2)$$

Cubing both sides and taking expectations results in

$$E(Z_j^3) = \rho_X^3 E(Z_{j-1}^3) + E(\tilde{t}_j^3) (1-\rho_X^2)^{3/2}$$

$$\text{for } E(\tilde{t}_j Z_{j-1}^2) = E(\tilde{t}_j^2 Z_{j-1}) = 0$$

because $\tilde{t}_j$, Z_{j-1} are uncorrelated, and $E(\tilde{t}_j^{\,m}\, Z_{j-1}^{\,n}) = E(\tilde{t}_j^{\,m})E(Z_{j-1}^{\,n}) = 0$, where m, n take the value of 1,2 but not simultaneously.

Therefore,

$$E(\tilde{t}_j^{\,3}) = \frac{E\,(Z_j^{\,3}) - \rho_X^{\,3}\, E\,(Z_{j-1}^{\,3})}{(1 - \rho_X^{\,2})^{3/2}} \qquad (1.3)$$

or, equivalently,

$$\gamma(\tilde{t}_j) = \frac{\gamma(X_j) - \rho_X^{\,3}\, \gamma(Y_{j-1})}{(1 - \rho_X^{\,2})^{3/2}} \qquad (1.4)$$

Thus the skewness of the independent Variable can be calculated. It is maintained by using some technique for generating random values from a gamma distribution with mean zero, variance unity and coefficient of skewness $\gamma(\tilde{t}_j)$, as given above.

APPENDIX 2

A TWO-VARIATE MODEL

(a) Preserving the Appropriate Correlations

Having set up equations of the form given by Eq.(1.1), illustrated in Table 5, the autocorrelation coefficient between inflows in a given month and the previous month in the Yarmouk is preserved.

However, in order to maintain the cross-correlation, as well as the coefficient of skewness for that particular month, a modification of the equation is deemed necessary.

A standard autoregressive model is of the form (as before) is

$$Y_j = \mu_{Y_j} + \rho_{(Y_j)}(Y_{j-1}-\mu_{Y_{j-1}}) \cdot \frac{\sigma_{Y_j}}{\sigma_{Y_{j-1}}} + \tilde{t}_j \; (1-\rho_{(Y_j)}{}^2)^{\frac{1}{2}} \sigma_{Y_j} \qquad (2.1)$$

where $\rho_{(Y_j)}$ gives the correlation between Y_j and Y_{j-1} and where $\tilde{t}_j$ is - typically - an independent gamma random variable with zero mean, unit variance and coefficient of skewness $\gamma(\tilde{t}_j)$ as found in Equation (1.4). Typically, because three months given by the empirical data follow normal distributions, and consequently have zero coefficient of skewness.

With the object of preserving the correlation between inflows into the Kinneret in a particular month (X_j) and inflows into Yarmouk for that month (Y_j), Eq. (2.1) can be modified by dropping the condition of independence upon $\tilde{t}_j$, and so $E(X_j.\tilde{t}_j)$ must be chosen in such a way that the correlation between X_j and Y_j will be maintained under generation.

Thus, multiplying Eq. (2.1) by X_j, and taking expectations, we have that

$$E(X_jY_j) = \mu_{Y_j} \cdot E(X_j) + \rho_{(Y_j)}E[X_j(Y_{j-1}-\mu_{Y_{j-1}})] \frac{\sigma_{Y_j}}{\sigma_{Y_{j-1}}} + E(X_jt_j)(1-\rho_{(Y_j)}{}^2)^{\frac{1}{2}}\sigma_{Y_j} \qquad (2.2)$$

Using the definition $E(X_jY_j) - E(X_j)E(Y_j) = \text{Cov}(X_j,Y_j)$ (2.3)

and $\mu_{Y_j} = E(Y_j)$

and where we have replaced $\tilde{t}_j$ (an independent random variable) by t_j, Eq. (2.2) can be put in the form:

$$\mathrm{Cov}\ (X_j,Y_j) = \rho(Y_j)\ \mathrm{Cov}(X_j,Y_{j-1})\ \frac{\sigma_{Y_j}}{\sigma_{Y_{j-1}}} + E(X_j.t_j)\ (1-\rho(Y_j)^2)^{1/2}\ \sigma_{Y_j} \quad (2.4)$$

Now, by definition, $\mathrm{Cov}\ (X_j,Y_j) = \sigma_{X_j}\ \sigma_{Y_j}\ \mathrm{Corr}\ (X_j,Y_j)$ (2.5)

where $\mathrm{Corr}\ (X_j,Y_j)$ denotes the correlation between X_j and Y_j.

Substituting, for the sake of parsimony, $\rho_{(X_jY_j)}$ for $\mathrm{Corr}\ (X_j,Y_j)$,

Eq. (2.4), using correlation coefficients, becomes

$$\rho_{(X_j\ Y_j)}\ \sigma_{X_j} = \sigma_{X_j}\rho(Y_j)\rho_{(X_jY_{j-1})} + E(X_jt_j)\ (1-\rho(Y_j)^2)^{1/2} \quad (2.6)$$

Therefore, the relationship between X_j and t_j must satisfy Eq. (2.6).

Rearranging the equation we find that

$$E(X_j.t_j) = \sigma_{X_j} \left\{ \frac{[\rho(X_j\ Y_j) - \rho_{(Y_j)}\rho(X_j\ Y_{j-1})]}{(1-\rho_{(Y_j)}{}^2)^{1/2}} \right\} \quad (2.7)$$

Since t_j is a variable with mean zero, and variance unity, the definition given by Eq. (2.3) is modified to read

$$\mathrm{Cov}\ (X_j,t_j) = E(X_j\ t_j)$$

and, according to the definition of $\mathrm{Corr}\ (X_j,t_j)$

$$\mathrm{Corr}\ (X_j,t_j) = \mathrm{Cov}\ (X_j,t_j)/\sigma_{X_j}.1$$

Denoting this correlation by R, and using Eq. (2.7) R is defined as

$$R = \frac{\rho_{(X_jY_j)} - \rho(Y_j)\ \rho_{(X_jY_{j-1})}}{(1-\rho(Y_j)^2)^{1/2}}\ . \quad (2.8)$$

It follows that if R takes the value above, then the cross-correlation between X_j and Y_j will be preserved.

A linear regression between t_j and X_j can therefore be set up in the form:

$$t_j = \frac{R}{\sigma_{X_j}} \cdot (X_j-\mu_{X_j}) + \tilde{S}_j\ (1-R^2)^{1/2}\ , \quad (2.9)$$

where $\tilde{S}_j$ is an independent random variable on (0,1).

Substituting this value for t_j in Eq. (2.1) it might be thought that the Lag 1 correlation, the cross-correlations, and the lagged cross-correlations (viz. Corr (X_j, Y_{j-1})), would be maintained. However, the new definition of t_j introduces a spurious factor into the correlation between Y_j and Y_{j-1}. This is due to the fact that because of the correlation between X_j and Y_{j-1}, there is necessarily a correlation between t_j defined by Eq. (2.9) and Y_{j-1} which - in order to preserve the correlation between Y_j and Y_{j-1} - should be zero.

This can be explained directly from the equations as follows. Multiply Eq. (2.1) by Y_{j-1} and take expectations:

$$E(Y_j Y_{j-1}) = \mu_{Y_j} \mu_{Y_{j-1}} + \rho(Y_j)\, E[Y_{j-1}(Y_{j-1} - \mu_{Y_{j-1}})] \frac{\sigma_{Y_j}}{\sigma_{Y_{j-1}}} +$$
$$+ E(t_j Y_{j-1})\, (1-\rho(Y_j)^2)^{\frac{1}{2}}\, \sigma_{Y_j} \qquad (2.10)$$

Noting that $\sigma_{Y_{j-1}}{}^2 = \mathrm{Var}\,(Y_{j-1})$ and by definition

$$\mathrm{Var}\,(Y_{j-1}) = E[Y_{j-1}(Y_{j-1} - \mu_{Y_{j-1}})],$$

then, if $E(t_j \cdot Y_{j-1}) = 0$, Eq. (2.10) would reduce to:

$$E(Y_j\, Y_{j-1}) - \mu_{Y_j} \mu_{Y_{j-1}} = \rho(Y_j)\, \sigma_{Y_j} \sigma_{Y_{j-1}} \qquad (2.11)$$

A similar expression to Eq. (2.3) defines Cov (Y_j, Y_{j-1}), whereby Eq. (2.11) becomes

$$\mathrm{Cov}\,(Y_j, Y_{j-1}) = \rho(Y_j)\, \sigma_{Y_j} \sigma_{Y_{j-1}},$$

Now, $\rho(Y_j)\sigma_{Y_j} \sigma_{Y_{j-1}}$ is in fact the definition of Cov (Y_j, Y_{j-1}) (compare to the definition given in Eq. (2.5)) and consequently the Lag-one correlation between Y_j and Y_{j-1} is maintained.

However, with t_j defined in Eq. (2.9), $E(t_j Y_{j-1})$ is in fact not zero, since, multiplying it through by Y_{j-1} and taking expectations we find that

$$E(Y_{j-1} \cdot t_j) = \frac{R}{\sigma_{X_j}} E\,[Y_{j-1}(X_j-\mu_{X_j})] + E(\tilde{S}_j\ Y_{j-1})\cdot(1-R^2)^{\frac{1}{2}} \qquad (2.12)$$

$\tilde{S}_j$ is an independent random variable distributed on (0,1) and thus

$$E(\tilde{S}_j\ Y_{j-1}) = E(\tilde{S}_j)\ E(Y_{j-1}) = 0 \cdot E(Y_{j-1}) = 0$$

But, $E[Y_{j-1}(X_j-\mu_j)] = \text{Cov}\ (Y_{j-1},X_j) = \rho(X_j Y_{j-1}) \cdot \sigma_{X_j}\ \sigma_{Y_{j-1}}$,

and therefore, $E(Y_{j-1}t_j) = R\ \rho(X_j Y_{j-1})\ \sigma_{Y_{j-1}}$, which is a non-zero quantity as claimed above. To prevent this spurious correlation, t_j must be defined as a variable, in such a way that $E(t_j X_j) = R$ (as above), but with $E(t_j \cdot Y_{j-1}) = 0$.

This can be achieved by selecting coefficients α_1, α_2 in Eq. (2.13) (below) that satisfy these relationships.

Consider a linear model of the form:

$$t_j = \frac{\alpha_1}{\sigma_{X_j}} (X_j-\mu_{X_j}) + \frac{\alpha_2}{\sigma_{Y_{j-1}}} \cdot (Y_{j-1}-\mu_{Y_{j-1}}) + (1-MC)^{\frac{1}{2}}\,\tilde{S}_j \qquad (2.13)$$

where MC represents the multiple correlation coefficient, denoting the reduction to the variance of t_j by the addition of X_j and Y_{j-1} to the model. To standardize the variables a substitution can be made as follows:

$$U_j = \frac{X_j-\mu_{X_j}}{\sigma_{X_j}}, \qquad V_{j-1} = \frac{Y_{j-1}-\mu_{Y_{j-1}}}{\sigma_{Y_{j-1}}}$$

Thus, U_j and V_{j-1} are distributed with zero mean and unit variances. Now Eq. (2.13) can be rewritten as

$$t_j = \alpha_1\ U_j + \alpha_2\ V_{j-1} + \tilde{S}_j\ (1-MC)^{\frac{1}{2}} \qquad (2.14)$$

In order to solve α_1, α_2 and MC, Eq. (2.14) is multiplied by U_j and V_{j-1} respectively, and expectations are taken, resulting in:

$$\left.\begin{aligned} E(U_j t_j) &= \alpha_1 E(U_j^2) + \alpha_2 E(U_j V_{j-1}) + E(U_j \tilde{S}_j)\ (1-MC)^{\frac{1}{2}} \\ E(V_{j-1} t_j) &= \alpha_1 E(U_j V_{j-1}) + \alpha_2 E(V_{j-1}^2) + E(\tilde{S}_j V_{j-1})\ (1-MC)^{\frac{1}{2}} \end{aligned}\right\} \quad (2.15)$$

From the above discussion we can summarize as follows:

(1) $E(U_j t_j) = R$ and $E(V_{j-1} t_j) = 0$

(2) $E(U_j^2) = \text{Var}\ (U_j^2) = 1$ and $E(V_{j-1}^2) = \text{Var}\ (V_{j-1}) = 1$

(3) $E(U_j \tilde{S}_j) = E(U_j)\ E(\tilde{S}_j)$ and $E(V_{j-1} \tilde{S}_j) = E(V_{j-1})\ E(\tilde{S}_j)$

for $\tilde{S}_j$ is independent of U_j and of V_j.

And so $E(U_j \tilde{S}_j) = E(V_{j-1} \tilde{S}_j) = 0$

and (4) $E(U_j V_{j-1}) = \text{Corr}(U_j, V_{j-1}) = \rho_{(UV)}$ (say)

Using these properties, the set of equations given by Eq. (2.15) can be rewrtitten as follows:

$$\left.\begin{aligned} R &= \alpha_1 + \alpha_2\ \rho_{(UV)} \\ 0 &= \alpha_1\ \rho_{(UV)} + \alpha_2 \end{aligned}\right\} \quad (2.16)$$

Therefore, $\alpha_1 = \dfrac{R}{1 - \rho_{(UV)}^{\ 2}}$

and $\alpha_2 = \dfrac{-\ R\ \rho(UV)}{1 - \rho_{(UV)}^{\ 2}}$

Thus if α_1 and α_2 are chosen in this manner $E(U_j t_j)$ and $E(V_j t_j)$ will be preserved under generation.

In order to evaluate the multiple correlation coefficient, multiply Eq. (2.14) by $\tilde{S}_j$, and take expectations:

$$E(t_j \tilde{S}_j) = E\ \{(\alpha_1 U_j + \alpha_2 V_{j-1} + \tilde{S}_j\ (1-MC)^{\frac{1}{2}})\ \tilde{S}_j\} \quad (2.17)$$

Since $\tilde{S}_j$ is independent of U_j and V_{j-1} and $E(\tilde{S}_j) = 0$ this reduces to

$$E(t_j\tilde{S}_j) = E(\tilde{S}_j^{\,2})\ (1-MC)^{\frac{1}{2}} .$$

Also, $E(\tilde{S}_j^{\,2})=\text{Var}\ (\tilde{S}_j) = 1$,

$$\text{then}\quad E(t_j\tilde{S}_j) = (1-MC)^{\frac{1}{2}} \tag{2.18}$$

Alternatively, Eq. (2.17) can also be expanded by substituting for $\tilde{S}_j$, resulting in

$$E(t_j\tilde{S}_j) = E\left\{\frac{t_j\ (t_j-\alpha_1 U_j-\alpha_2 V_{j-1})}{(1-MC)^{\frac{1}{2}}}\right\}$$

$$= \frac{1}{(1-MC)^{\frac{1}{2}}}\ [E(t_j^{\,2}) - \alpha_1\ E(t_jU_j) - \alpha_2\ E(t_jV_{j-1})]$$

Substituting R for $E(t_jU_j)$, 0 for $E(t_jV_{j-1})$ and 1 for $E(t_j^{\,2})$ we find that

$$E(t_j\tilde{S}_j) = \frac{1}{(1-MC)^{\frac{1}{2}}}\ (1 - \alpha_1 R) \tag{2.19}$$

Equating Eq. (2.18) with Eq. (2.19) through $E(t_j\tilde{S}_j)$, we obtain

$$(1-MC)^{\frac{1}{2}} = \frac{1}{(1-MC)^{\frac{1}{2}}}\ (1 - \alpha_1 R)\ ;$$

and so $MC = \alpha_1 R$

Transforming back to the original variables, and substituting for α_1, α_2 and MC, Eq. (2.13) becomes:

$$t_j = \frac{R}{\sigma_{X_j}\ [1-\rho^2(X_jY_{j-1})]}\ (X_j-\mu_{X_j}) - \frac{R\ \rho(X_j, Y_{j-1})}{\sigma_{Y_{j-1}}\ [1-\rho^2(X_jY_{j-1})]}\ (Y_{j-1}-\mu_{Y_{j-1}}) + $$

$$+ \left[1 - \frac{R^2}{1-\rho^2(X_jY_{j-1})}\right]^{\frac{1}{2}}\ \tilde{S}_j \tag{2.20}$$

with R evaluated in Eq. (2.8).

Thus Eq. (2.20) combined with Eq. (2.1) preserves the appropriate correlations, viz. Corr (X_j,Y_{j-1}), Corr (Y_j,Y_{j-1}), Corr (X_j,Y_j).
It should be noted that the result of Eq. (2.20) being substituted into Eq. (2.1) is in fact the common regression equation with two independent variables. More precisely, the coefficients β_1 and β_2 (below) are chosen in much the same way as α_1, α_2 given by Eq. (2.13), where Y_j is calculated from an equation of the form:

$$Y_j = \mu_{Y_j} + \frac{\sigma_{Y_j}}{\sigma_{X_j}} \beta_1 (X_j-\mu_{X_j}) + \frac{\sigma_{Y_j}}{\sigma_{Y_{j-1}}} \beta_2 (Y_{j-1}-\mu_{Y_{j-1}}) +$$

$$+ \tilde{S}_j (1-\beta_1\rho_{(X_j\,Y_j)} - \beta_2\rho_{(Y_j\,Y_{j-1})})^{\frac{1}{2}} \sigma_{Y_j} \qquad (2.21)$$

β_1 and β_2 provide for the preservation of the appropriate correlations, means and standard deviations.

It can be shown that the two sets of equations, namely Eq.(2.1) and Eq. (2.20) together, and Eq. (2.21) are both necessary and sufficient in preserving the appropriate parameters, therefore they must be identical.

(b) <u>Preserving the Coefficient of Skewness</u>

However, the advantage of the above step-wise analysis is that it is possible to find explicitly the coefficient of skewness of $\tilde{S}_j$, and thereby to maintain that of t_j and consequently of Y_j, under large-sample generation.

Calculating the coefficient of skewness of a random variable within the framework of a multiple regression (as given in Eq. (2.21) leads to difficulties:

By making a substitution in order to standardize the variables, viz.

$$U_j = \frac{X_j-\mu_{X_j}}{\sigma_{X_j}} \quad \text{and} \quad V_j = \frac{Y_j-\mu_{Y_j}}{\sigma_{Y_j}}$$

Equation (2.21) becomes:

$$V_j = \beta_1 U_j + \beta_2 V_{j-1} + \tilde{S}_j \, [1-\beta_1\rho(V_j\,U_j) - \beta_2\rho(V_j\,V_{j-1})]^{\frac{1}{2}} \qquad (2.22)$$

In order to find the coefficient of skewness of $\tilde{S}_j$ and by so doing maintain that of V_j, both sides of Eq. (2.22) are cubed and expectations taken:

$$E(V_j^3) = \beta_1^3\, E(U_j^3) + \beta_2^3\, E(V_{j-1}^3) + E(\tilde{S}_j^3)\, [1-\beta_1\rho(V_jU_j) - \beta_2\rho(V_j\, V_{j-1})]^{\frac{1}{2}}$$

$$+ 3\,\beta_1^2\beta_2\, E(U_j^2V_{j-1}) + 3\,\beta_1\beta_2^2\, E(U_jV_{j-1}^2) \qquad (2.23)$$

The other terms are zero because of the independency of $\tilde{S}_j$.

Evaluation of the covariance terms can only be done through the moments of a joint gamma probability distribution. However in the general case a joint gamma distribution has not been evolved (5), and so the moments given above can only be found as approximations (6) and, consequently, the coefficient of skewness of the random variable can not be expected to be maintained.

By using Eq. (2.20) with Eq. (2.1) it is possible to circumvent the difficulties arising from the use of Eq. (2.21). Eq. (2.20) can be rewritten as

$$t_j = \frac{\alpha_1}{\sigma_{X_j}}(X_j-\mu_{X_j}) + \frac{\alpha_2}{\sigma_{Y_{j-1}}}(Y_{j-1}-\mu_{Y_{j-1}}) + (1-\alpha_1\, R)^{\frac{1}{2}}\,\tilde{S}_j \qquad (2.24)$$

where $\alpha_1 = \dfrac{R}{1-\rho^2_{(X_jY_{j-1})}}$ and $\alpha_2 = \dfrac{-R\,\rho_{(X_jY_{j-1})}}{1-\rho^2_{(X_jY_{j-1})}}$

with R defined in Eq. (2.8).

Eq. (2.24) is equivalent to:

$$t_j - \frac{\alpha_2}{\sigma_{Y_{j-1}}}(Y_{j-1}-\mu_{Y_{j-1}}) = \frac{\alpha_1}{\sigma_{X_j}}(X_j-\mu_{X_j}) + (1-\alpha_1\, R)^{\frac{1}{2}}\,\tilde{S}_j \qquad (2.25)$$

Cubing both sides of Eq. (2.25) and taking expectations, results in

$$\gamma(t_j) - \alpha_2^3\,\gamma(Y_{j-1}) = \alpha_1^3\,\gamma(X_j) + \gamma(S_j)\,(1-\alpha_1\, R)^{3/2}$$

where $\gamma(Z_j)$ is the coefficient of skewness of Z_j, given by

$$\gamma(Z_j) = \frac{E(Z_j-\mu_j)^3}{(\sigma_{Z_j})^{3/2}}$$

The terms $E[t_j(Y_{j-1}-\mu_{Y_{j-1}})^2]$, $E[t_j^2(Y_{j-1}-\mu_{j-1})]$

and $E[S_j^2(X_j-\mu_{X_j})]$, $E[S_j(X_j-\mu_{X_j})^2]$

vanish, for t_j is uncorrelated with Y_j as is $\tilde{S}_j$ with X_j, and t_j, $\tilde{S}_j$ are standardized gamma random variables.

Therefore,

$$\gamma(\tilde{S}_j) = \frac{\gamma(t_j) - \alpha_2^3\ \gamma(Y_{j-1}) - \alpha_1^3\ \gamma(X_j)}{(1 - \alpha_1\ R)^{3/2}} .$$

Such a value of $\tilde{S}_j$ would ensure that the skewness of Y_j would be maintained under generation.

APPENDIX 3

CONVERTING AN ANNUAL STANDARD DEVIATION INTO MONTHLY DEVIATIONS

An artificial rain factor - which is thought to increase the annual values by factors of 0.1 and 0.2, with estimated annual standard deviations - needs to be included in the monthly generation scheme.

σ_1, σ_2, the monthly standard deviations of the increase, are calculated using the identity below:

$$\begin{aligned} \text{Var}\ \{X(1.1 + 0.051\ t)\} &= \text{Var}\ \{\sum_{i=1}^{12} X_i\ (1.1 + \sigma_1 t_i)\} \\ \text{Var}\ \{X(1.2 + 0.056\ t)\} &= \text{Var}\ \{\sum_{i} X_i\ (1.1 + \sigma_2 t_i)\} \end{aligned} \tag{3.1}$$

where $X = \sum_{i=1}^{12} X_i$ and t, t_i are independent normal random variables on (0.1).

Consider $\text{Var}\ \{X(a + bt)\} = \text{Var}\ \{\sum_i X_i\ (a + \sigma\, t_i)\}$ (3.2)

Now, $\text{Var}\ \{X(a + bt)\} = a^2\ \text{Var}\ (X) + b^2\ \text{Var}\ (Xt) + 2ab\ \text{Cov}\ (X,Xt)$ (3.3)

This expression can be expanded term by term as follows:

$$\begin{aligned} \text{Var}\ (Xt) &= E(Xt)^2 - E^2\ (Xt), \text{ by definition} \\ &= E(X^2)E(t^2) - [E(X).E(t)]^2 \text{ for X and t are independent} \\ &= E(X^2).1 - 0 = E(X^2) \end{aligned}$$

Also, $\text{Cov}\ (X,Xt) = E(X.Xt) - E(X).E(Xt)$

$$= E(X^2)E(t) - [E(X)]^2\ E(t) = 0$$

Therefore, it follows from Eq. (3.3) that

$$\text{Var}\ \{X(a + bt)\} = a^2\ \text{Var}\ (X) + b^2\ E(X^2) \tag{3.4}$$

The right hand side of Eq. (3.2) can be expressed as follows:

$$\mathrm{Var}\{\sum_{i=1}^{12} X_i\,(a + \sigma t_i)\} = \mathrm{Var}\,\{a \sum_j X_i + \sigma\sum_j X_j t_j\}$$

$$= a^2\,\mathrm{Var}\,(\sum_i X_i) + \sigma^2\,\mathrm{Var}\,(\sum_j X_j t_j) + 2\,a\sigma\,\mathrm{Cov}\,(\sum_i X_i, \sum_j X_j t_j) \qquad (3.5)$$

By denoting

$$(A) = \mathrm{Var}\,(\sum_i X_i)$$

$$(B) = \mathrm{Var}\,(\sum_j X_j t_j)$$

$$(C) = \mathrm{Cov}\,(\sum_i X_i, \sum_j X_j t_j)$$

we find that

$$(A) = \mathrm{Var}\,(X) \qquad (3.6)$$

$$(B) = \sum_j \mathrm{Var}\,(X_j t_j) + 2 \sum_{i<j}\sum \mathrm{Cov}\,(X_i t_i, X_j t_j) \qquad (3.7)$$

The terms in Eq. (3.7) can be expanded as follows:

$$\sum_j \mathrm{Var}\,(X_j t_j) = \sum_j \{E(X_j t_j) - E^2\,(X_j t_j)\}$$

$$= \sum_j \{E(X_j^2)E(t_j^2) - [E(X_j)E(t_j)]^2\}$$

$$= \sum_j E(X_j^2).1 - 0 = \sum_j E(X_j^2)$$

Also in Eq. (3.7)

$$2 \sum_{i<j}\sum \mathrm{Cov}\,(X_i t_i, X_j t_j) = 2 \sum_{i<j}\sum \{E(X_i t_i . X_j t_j) - E(X_i t_i)E(X_j t_j)\}$$

$$E(X_i t_i . X_j t_j) = E(X_i X_j).E(t_i t_j) = 0 \text{ for } i \neq j$$

and $E(X_i t_i) = E(X_i)\;E(t_i) = 0$

Thus $(B) = \sum_j E(X_j^2)$.

$$(C) = E[\sum_i X_i \sum_j X_j t_j] - E[\sum_i X_i]\, E[\sum_j X_j t_j]$$

$$= E[\sum_i X_i^2 t_i + \sum_{\substack{i\ j \\ i \neq j}} X_i X_j t_j] - E(X) \sum_j E(X_j t_j)$$

$$= 0 - 0$$

Therefore using Eq. (3.4) and Eq. (3.5), Eq. (3.2) can be rewritten as

$$a^2 \text{ Var }(X) + b^2 E(X^2) = a^2 \text{ Var }(X) + \sigma^2 \Sigma E(X_j^2) + 0$$

$$\text{Therefore } \sigma^2 = \frac{b^2 E(X^2)}{\sum_j E(X_j^2)}$$

$$\text{Now, Var }(X) = E(X^2) - [E(X)]^2$$

$$\text{and Var }(X_j) = E(X_j^2) - E(X_j)$$

$$\text{Therefore } \sigma^2 = \frac{b^2 \{\text{Var}(X) + [E(X)]^2\}}{\sum_i [\text{Var }(X_i) + \mu_i^2]}$$

where μ_i = mean of month i

σ_1 and σ_2 can be calculated for the two values of b

For b = 0.051, $\sigma_1 = 0.582$

For b = 0.056, $\sigma_2 = 0.641$

```
 SREC    SIDEX   SUBSC   SUBIN

REAL CONSTANTS
  .250000E 00=0172    .200000E 01=0174    .300000E 01=0176    .100000E 01=0178    .600000E 01=017A

INTEGER CONSTANTS
     2=017C      1=017D      12=017E      24=017F      3=0180

CORE REQUIREMENTS FOR SSKEW
 COMMON        0  VARIABLES     370  PROGRAM     368

RELATIVE ENTRY POINT ADDRESS IS 0187 (HEX)

END OF COMPILATION

// DUP

*DELETE              SSKEW
CART ID 0133   DB ADDR  54F9   DB CNT   001A

*STORE       WS  UA  SSKEW            0133
CART ID 0133   DB ADDR  5831   DB CNT   001A

// FOR
*IOCS(CARD)
*ONE WORD INTEGERS
*LIST ALL
*IOCS (1132 PRINTER)
*IOCS(DISK)
      DEFINE FILE 1(1801,8,U,IXX)
C
C  GENERATION OF DATA INTO KINNERET AND YARMOUK
C
      INTEGER S
      DIMENSION K(12),DEV(12),A(12,5),F(12),RHO(12),GAMMA(12),X(12),Z(12
     1),XX(12),KK(12),DYEV(12),B(12,3),ZY(12),C(12),G(12),XXX(12),Y(12)
     1,ANS(4),SMEAN(4)
      DO 53  I=1,4
   53 ANS(I)=0.
      ISW=1
C
C  READ IN THE DATA INTO KINNERET ONCE IT HAS BEEN REDUCED TO EQUATION FORM,
C    E.G.   X(J)=A(J,1)+A(J,2)X(J-1)+A(J,3)X(J-2)+T*XDEV(J)
C
C      K(J) IS NUMBER OF INPUT TERMS
C
      DO 8 J=1,12
      READ(2,200)K(J)
  200 FORMAT(I4)
      READ(2,201) DEV(J)
  201 FORMAT(F7.3)
      I=1
      READ(2,201)A(J,I)
   12 I=I+1
      IF(I-K(J))11,11,13
   11 READ(2,201)A(J,I)
      GOTO 12
C
```

APPENDIX 4: PROGRAMME FOR GENERATING SEQUENCE REPRESENTING INFLOWS INTO LAKE KINNERET AND YARMOUK RIVER

```
PAGE   1    10.12.73

// JOB    0112 0133

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0112        0112        0000
  0001        0133        0133        0004
                          0121        0001
                          0100        0002

V2 M09   ACTUAL 32K  CONFIG 32K

// DUP

// FOR
*LIST ALL
*ONE WORD INTEGERS
      SUBROUTINESSKEW(J,T,ISW)
C
C     TO MAINTAIN SKEWNESS OF YARMUK
C
      DIMENSION YAA(12),AS(24) ,PLIN(12),YSL(12),  ENTER(12),BS(24),
     1YBB(12),TKEW(12),YH(12),YHH(12),YGG(12),YG(24)
      GOTO(9,10),ISW
    9 READ(2,3)(TKEW(I),I=1,12)
    3 FORMAT(12F6.0)
      READ(2,2)(AS(I),I=1,24)
    2 FORMAT(6F10.0)
      READ(2,2) (BS(I),I=1,24)
      READ(2,2) (YG(I),I=1,24)
      READ(2,3)(PLIN(I),I=1,12)
      ISW=2
   10 GOTO(11,11,11,11,11,11,11,8 ,8,8,11,11),J
   11 ENTER(J)=PLIN(J)/0.25
      L=2*J-1
      YAA(J)=AS(L)-(AS(L)-AS(L+1))*ENTER(J)
      YBB(J)=BS(L)-(BS(L)-BS(L+1))*ENTER(J)
      YH(J)=YBB(J)-2.0/(TKEW(J)*YAA(J))
      YHH(J)=EXP(ALOG(YH(J))/3.)
      YGG(J)=YG(L)-(YG(L)-YG(L+1))*ENTER(J)
      YSL(J)=1.-(YGG(J)/6.)**2+(YGG(J)/6.)*T
      IF(YHH(J)-YSL(J)) 5,5,6
    5 TT=YAA(J)*(YSL(J)**3-YBB(J))
      GOTO 7
    6 TT=YAA(J)*(YHH(J)**3-YBB(J))
    7 T=TT
    8 RETURN
      END
      VARIABLE ALLOCATIONS
         YAA(R )=0016-0000      AS(R )=0046-0018  PLIN(R )=005E-0048   YSL(R )=0076-0060 ENTER(R )=008E-0078     BS(R )=00BE-0090
         YBB(R )=00D6-00C0   TKEW(R )=00EE-00D8     YH(R )=0106-00F0   YHH(R )=011E-0108   YGG(R )=0136-0120     YG(R )=0166-0138
          TT(R )=0168            I(I )=016C          L(I )=016D

      STATEMENT ALLOCATIONS
       3    =0181  2    =0184  9    =019D  10   =020F  11   =021F  5    =02BC  6    =02CD  7    =02DC  8    =02E0

      FEATURES SUPPORTED
       ONE WORD INTEGERS

      CALLED SUBPROGRAMS
       FEXP    FALOG   FADD    FSUBX   FMPY    FMPYX   FDIV    FLD     FLDX    FSTO    FSTOX   FSBR    FSBRX   FDVR    FAXI
```

```
C     FOR THOSE MONTHLY INPUTS THAT FOLLOW A GAMMA DISTRIBUTION WITH
C       SKEWNESS OF T)3.0 READ IN DATA FOR TRANSFORMATION
C
   13 GOTO(3,8,8,8,8,8,8,8,8,8,8,3),J
    3 READ(2,201)F(J)
      READ(2,201)RHO(J)
      READ(2,201)GAMMA(J)
    8 CONTINUE
C
C     READ DATA INTO YARMOUK IN SAME WAY AS KINNERET
C
      DO19CJ=1,12
      READ(2,200) KK(J)
      READ(2,201)DYEV(J)
      I=1
      READ(2,201)B(J,I)
   14 I=I+1
      IF(I-KK(J))15,15,190
   15 READ(2,201)B(J,I)
      GO TO 14
  19C CONTINUE
C
C   C(J) IS COEFF OF X(J)
C
      READ(2,202) (C(J),J=1,12)
  202 FORMAT(12F6.0)
C
C  GENEAETE VALUES INTO KINNERET
C
C   IX IS STARTING VALUE FOR GENERATING T
C
      IX=17142
      L=1
      DO9 N=1,2C0
      DO 9 J=1,12
      JK=1
      I=1
      II=C
      Z(J)=A(J,I)
  130 I=I+1
      IF(I-K(J)) 110,110,120
  110 II=II+1
      S=J-II
      IF(S-1)500,7 , 7
  500 S=12
      IF(N-1)4,4,7
    4 TOTAL=Z(J)+A(J,I)*27.984
      GOTO99
    7 TOTAL=Z(J)+A(J,I)*X(S)
   99 Z(J)=TOTAL
      GO TU 13C
  120 CALL GAUSS(IX,1.0,0.0,T)
C
C   DIVIDE DATA 3 WAYS (1)NORMAL (2)GAMMA LOW COEFF  (6,922)GAMMA HIGH
C     COEFF
C
      GOTO(2,6,1,1,922,1,1,1,1,1,1,2),J
    2 G(J)=(F(J)-RHO(J)**3*GAMMA(J))/(SQRT(1.-RHO(J)**2)**3)
```

```
      TT=(2./G(J))*(1.0+G(J)*T/6.0-G(J)**2/36.0)**3-2.0/G(J)
      GOTO  19
  922 AA=0.51924-(0.51924-0.50655)*138./250.
      BB=(1.14956-1.12311)*138./250.+1.12311
      H=BB    -2.0/(5.138*AA)
      HH=EXP(ALOG(H)/3.)
      GG=(4.30047-4.15577)*138./250.+4.15577
      SL=1-( GG   /6.)**2+( GG   /6.)*T
      IF(HH-SL) 930,930,931
  930 TT=  AA   *(SL**3-BB)
      GOTO 19
  931 TT=      AA*(HH**3-BB)
      GOTO 19
    6 H=1.25233-2./(6.05200*0.47545          )
      HH=EXP(ALOG(H)/3.)
      SL=1-(4.70984/6.)**2+(4.70984/5.)*T
      IF(HH-SL)330,330,331
  330 TT=0.47545*(SL**3-1.25233)
      GOTO  19
  331 TT=0.47545*( HH**3-1.25233)
   19 T=TT
    1 X(J)=Z(J)+T*DEV(J)
      IF(N-50)9,9,61
   61  ANS(JK)=ANS(JK)+X(J)
      JK=JK+1
      CALL GAUSS(IX,1.0,0.0,T)
      L=L+1
C
C    XXX(J)=INFLOW INTO KINNERET WITH ARTIFICIAL RAIN FACTOR OF 0.1
C
        IF(X(J)-0.  )613,614,614
  613 XXX(J)=X(J)*(0.9+0.582*T)*630./6..
      ANS(JK)=ANS(JK)+XXX(J)
      JK=JK+1
C
C     XX(J)=INFLOW INTO KINNERET WITH ARTIFICIAL RAIN FACTOR OF 0.2
C
      XX(J)=X(J)*(0.8+0.641*T)*690./678.
      ANS(JK)=ANS(JK)+XX(J)
      JK=JK+1
      GOTO 615
  614 XXX(J)=X(J)*(1.1+0.582*T)*622./630.
      ANS(JK)=ANS(JK)+XXX(J)
      JK=JK+1
      XX(J)=X(J)*(1.2+0.641*T)*678./690.
      ANS(JK)=ANS(JK)+XX(J)
      JK=JK+1
  615 I=1
      II=0
      ZY(J)=B(J,I)
   22 I=I+1
      IF(I-KK(J))20,20,21
   20 II=II+1
      S=J-II
      TTOT=ZY(J)+B(J,I)*Y(S)
      ZY(J)=TTOT
      GOTO 22
   21 CALL GAUSS(IX,1.0,0.0,T)
```

```
C
C    TO MAINTAIN COEFF OF SKEWNESS OF YARMOUK INFLOWS
C
      CALLSSKEW(J,T,ISW)
      Y(J)=ZY(J)+C(J)*X(J)+T*DYEV(J)
      IF(Y(J)-0.) 25,26,26
   25 Y(J)=0
   26 ANS(JK)=ANS(JK)+Y(J)
      WRITE(1'L)      X(J),XXX(J),XX(J),Y(J)
    9 CONTINUE
      DO 87 JK=1,4
   87 SMEAN(JK)=ANS(JK)/1800.
      WRITE(1'1)      (SMEAN(J),J=1,4)
      WRITE(3,88)     (SMEAN(J),J=1,4)
   88 FORMAT(1H0,'AVERAGE INFLOWS',5X,'KINNERET='F7.3,3X,'KINNERET+0.1=
     1'F7.3,3X,'KINNERET+0.2='F7.3,3X,'YARMOUK=',F7.3)
      DO 80 I=2,1801
      READ(1 'I)AX,AY,AZ,AW
      WRITE(3,81)AX,AY,AZ,AW
   81 FORMAT(1H ,10X,F5.1,10X,F5.1,10X,F5.1,10X,F5.1)
   80 CONTINUE
      CALL EXIT
      END
VARIABLE ALLOCATIONS
   DEV(R )=C01E-0008      A(R )=0096-0020      F(R )=00AE-0098    RHO(R )=00C6-0080 GAMMA(R )=00DE-00C8       X(R )=00F6-00E0
     Z(R )=C10E-00F8     XX(R )=0126-0110   DYEV(R )=013E-0128      B(R )=0186-0140    ZY(R )=019E-0188       C(R )=0186-01A0
     G(R )=01CE-01B8    XXX(R )=01E6-01D0      Y(R )=01FE-01E8    ANS(R )=0206-0200 SMEAN(R )=020E-0208  TOTAL(R )=0210
     T(R )=0212          TT(R )=0214          AA(R )=0216          BB(R )=0218           H(R )=021A          HH(R )=021C
    GG(R )=021E          SL(R )=0220        TTOT(R )=0222          AX(R )=0224          AY(R )=0226          AZ(R )=0228
    AW(R )=022A           K(I )=023F-0234     KK(I )=0248-0240      S(I )=024C         IXX(I )=024D           I(I )=024E
   ISW(I )=024F           J(I )=0250          IX(I )=0251           L(I )=0252           N(I )=0253          JK(I )=0254
    II(I )=0255

STATEMENT ALLOCATIONS
 200  =02A2  201  =02A4  202  =02A6  88   =02A9  81   =0208  53   =0300  12   =033E  11   =034F  13   =035D  3    =036D
 8    =0385  14   =03B6  15   =03C7  190  =03D5  130  =0422  110  =0433  500  =0445  4    =044F  7    =0467  99   =0486
 120  =0491  2    =04A7  922  =04F8  930  =0552  931  =055E  6    =056A  330  =05A0  331  =05AC  19   =05B6  1    =05BA
 61   =05CD  613  =0601  614  =0650  615  =069D  22   =0687  20   =06C8  21   =06FB  25   =0724  26   =072E  9    =0752
 87   =0768  80   =07C5

FEATURES SUPPORTED
 ONE WORD INTEGERS
 IOCS

CALLED SUBPROGRAMS
 GAUSS    FSQRT    FEXP     FALOG    SSKEW    FADD     FADDX    FSUB     FMPY     FMPYX    FDIV     FDIVX    FLD      FLDX     FSTO
 FSTOX    FSBR     FSBRX    FDVR     FAXI     FLOAT    CARDZ    PRNTZ    SRED     SWRT     SCOMP    SFIO     SIOFX    SIOIX    SIOF
 SUBSC    SDFIO    SDRED    SDWRT    SDCOM    SDFX     SDF

REAL CONSTANTS
  .000000E 00=025A   .279840E 02=025C   .100000E 01=025E   .200000E 01=0260   .600000E 01=0262   .360000E 02=0264
  .519240E 00=0266   .506550E 00=0268   .138000E 03=026A   .250000E 03=026C   .114956E 01=026E   .112311E 01=0270
  .513800E 01=0272   .300000E 01=0274   .430047E 01=0276   .415577E 01=0278   .125233E 01=027A   .605200E 01=027C
  .475450E 00=027E   .470984E 01=0280   .900000E 00=0282   .582000E 00=0284   .630000E 03=0286   .622000E 03=0288
  .800000E 00=028A   .641000E 00=028C   .690000E 03=028E   .678000E 03=0290   .110000E 01=0292   .120000E 01=0294
  .180000E 04=0296

INTEGER CONSTANTS
     1=0298      4=0299     12=029A      2=029B  17142=029C    200=029D      0=029E      3=029F     50=02A0   1801=02A1
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