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AN EXPRESSION OF THE CHINESE GENIUS.

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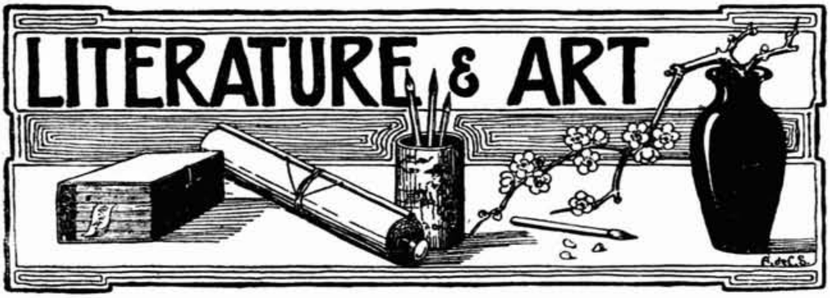

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AUTHOR'S NOTE.

"The Precious Mirror of the Four Elements" is a Chinese algebra by Chu Shih Chieh 朱世傑 *published in* 1303, *of which Dr. Chen Tsai Hsin, head of the Department of Mathematics in Yenching Ta Hsüeh, has made a complete translation, accompanied with numerous explanatory notes. I have been assisting in the work and the "Precious Mirror" itself is my authority.*

The road over which science has travelled from its very beginning to its present lofty height is a most fascinating one, both in its historic development and in its expression of the genius of a race. This is particularly true in the field of mathematics, which has entered so largely into the material, the intellectual, and the spiritual phases of the development of civilization.

Primitive man is concerned above all with that which deals with his daily life. Before a written language is developed the rude savage keeps his simple records on a Totem pole, a series of knots tied in a string, or by means of pebbles collected along the shore. Thus in the beginnings of all civilizations there is a conception of number—the one universal language.

A year ago the world was startled by the discovery of the Tomb of Tutankhamen with its wonderful treasures. About fifty years ago Egypt yielded another treasure, the Ahmes papyrus,* now preserved in the

* Deciphered by Eisenlohr in 1877.

Rhind Collection in the British Museum. From the treasures in royal tombs we see the culmination of Egyptian civilization: from this papyrus we see its beginnings. This papyrus, written about 1,700 B.C., is based upon earlier papyri written about 2,300 B.C. Ahmes, the priest, designates it as " Containing the Knowledge of All Dark Things." From it we learn that the Egyptians had developed the rudiments of Astronomy, by which they calculated the dates of their religious festivals, and that they knew the rudiments of geometry, by which they resurveyed the Nile districts after their yearly inundations and by which they determined the orientation of their pyramids and temples. Thus their astronomy and geometry, so far as we know, were cultivated for their practical ends. Breasted claims that their calendar year of three hundred and sixty five days, introduced in 4241 B.C., is the " earliest fixed date in the history of the world as known to us."*

It is to the Greeks, the heritors of the Egyptians, that we turn for the scientific development of geometry. In Egypt, mathematical knowledge was largely confined to the priesthood, the leisured class. In Greece it was no longer under the sway of the priestly class, and the Greeks, thus freed and unrestrained, took a higher and broader view and became the great geometers of the world. They were not the great sculptors of the world because the Pyrrean Hills were imbedded with beautiful marble, this was only a fortuitous medium for the expression of their thought. While the gladiators in the amphitheater and the heroes of the Olympic games may have given inspiration, they were the great sculptors of the world because they loved form. Elie Faure says, " The Greeks, even to the days of their saddest decline, loved these forms for themselves."†

Greek sculpture, in its dealing with three dimensional space, was born of Greek geometry. Their principles of geometry were a part of their philosophy of life. The Pythagorean school was more mathematical than philosophical. It made geometry a theoretical science. Plato placed his emphasis upon geometry as a training of the mind when he had inscribed above the entrance to his lecture room, " Let no one unacquainted with geometry enter here ; " when he said to a student who knew no geometry, " Depart from us for thou hast not the grip of philosophy," and when he replied on being questioned regarding the occupation of the Deity, " The Deity geometrizes continually."

The exquisite lines of the Greek vase, so long the admiration of the world, were based upon geometric laws. Jay Hambridge, in his book on Dynamic Symmetry, shows that the Greeks employed geometry in their all over proportion, the ratio between height and width being based upon root rectangles ($\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$) and other rectangles derived from these. Dr. Caskey, Curator of Classical Antiqities in the Boston Museum of Fine Arts, in his book on the " Geometry of the Greek Vase," has shown that the *amount of error* in the vases of the Museum, measured by the laws of dynamic symmetry, averages less than a millimeter. The Greek genius was nowhere more fully expressed than in geometry.

* Breasted : A History of Egypt, page 14.

† Elie Faure : Ancient Art, page 293.

The genius of the Romans—the great road builders, the great law givers—turned to algebra rather than geometry. Theoretic geometry made no appeal to them. They developed algebra largely upon the basis of what they inherited from the Arabs. Algebra is but another name for Italian Algorism—from Al-Khowarizmi—an abbreviation for Mohammed ibn Al-Khowarizmi, introduced by the Arabs in their migrations into Spain. Thus we inherited the Arabic system of notation. But whence the Arabic system ? From India. If we go back nearer the source, our system is not the Arabic system but the Hindoo system. Whence the Hindoo system ? Did it originate in India or in China ? Did the Indians and the Chinese develop their system independently or was one influenced by the other ? These are questions yet to be answered. It may be, due to the destruction of original sources, that the solution will have to depend wholly upon internal evidences.

We know, however, that we must look to the Far East for the early development of number system and algebra. The philosophy of the East was concerned with number rather than with form. While the Chinese, like other ancient peoples, early discovered certain astronomical phenomena, their greatest achievement was in arithmetic and algebra. They reached the summit of their achievement in algebra near the close of the thirteenth century, when Chü Shih Chieh published his " Szu Yüan Yü Chien," or " The Precious Mirror of the Four Elements."

Toward the end of the seventeenth century a Roman Catholic missionary presented to the Emperor Kang Hsi an algebra translated from European sources. The Emperor gave it, for inspection, to Mei Wên Ting 梅文鼎, one of the prominent mathematicians of the Empire, who pronounced it exactly the same as the old Chinese method known in the thirteenth century or earlier. He also claimed that the Western World derived the science from the East and that algebra meant " Came from the East."

Chu was widely known during his time as a teacher of mathematics, having travelled throughout the country teaching. The number of his students increased daily and he was prevailed upon to write a book to reveal the secrets of his work. " The Precious Mirror," first published in 1303, consists of the solutions of equations up to the thirteenth degree and of simultaneous equations in two, three, and four unknown quantities —or elements—these elements being *t'ien*, *ti*, *jên* and *wu*,—heaven, earth, man, and thing. These problems were solved by means of a calculating board and calculating rods made of bamboo, those coloured red representing positive quantities and those black, negative. Two hundred and seventy one rods commonly formed a set. Their origin will probably never be known, though some have placed it at about 3300 B.C.

I shall not go into the intricacies of the methods of solution. The difficulties, perhaps, can no better be pointed out than by quoting from the introduction to Chu's work itself. Chien Chin Shih Mo Ju says, " His method of solving equations is by putting the *yuan ch'i* (element of void), at the centre, the element *t'ien* at the bottom, the element *ti* at the left, the element *jên* at the right, and the element *wu* at the top ; by moving the positive and negative terms from the top to the bottom, from the right to the left, by interchanging and alternating their positions, and by many other different ways of arrangement of the terms . . . . . It is profoundly wonderful. It extends the principles which were founded by the ancient scholars. By the concentration of many elements into one, by controlling the *san ts'ai* (three talents) under the great extremity, and by multiplication and division, addition and subtraction, it reaches out to the great depth and the far distance. It is a book of science in itself.

"Now the science of mathematics is considered very important and an examination of the subject will gradually appear. The book of the master, therefore, will be of great benefit to the people of the world. The knowledge for investigation, the development of intellectual power, the way of controlling the kingdom and of ruling even the whole world, can be obtained by those who are able to make good use of the book. Ought not those who have great desire to be learned take this with them and study it with great care ?"

Tsu Yi Chi Hsien Fu writes, "He (Chu) has travelled through the country and at present is sojourning in Kuangling. People, like clouds, come from the four directions to meet at his gate in order that they may learn from him." . . . . "By the aid of geometric figures he explains their relations (*t'ien, ti, jên* and *wu*). By moving the expression from top to bottom, from right to left, by applying multiplication and division, by various methods in arranging the terms, by assuming the unreal for the real, by using the imaginary for the true, by using the signs, positive and negative, by keeping some and eliminating others, and then changing the position of the sticks, and by attacking from the front or from one side, as shown in the four examples, he finally works out the expression of evolution in a profound yet natural manner. When I was asked to write an introduction I read it through with great care and found that there were many things that I had never seen or heard of before. By not using 'yet' it is used ; by not using a number the number required is obtained. Hence I know that existing quantities come from non-existing quantities. This profound work is therefore exceedingly progressive as compared with the work of ancient mathematicians. Those who have an interest in the subject may prove my words by working out the problems in this book, thus finding the truth of my statements."

Before discussing the "Precious Mirror" further, let us consider the material Chu had at his disposal. The origin of Chinese mathematics is veiled in myth. Tradition tells that the ancient sage Fu Hsi (2852 B.C.) saw a dragon horse emerge from the Yellow River with a magic square upon its back. Yu, the first emperor of the Hsia Dynasty, is said to have observed the *Shu* (book) on the back of the sacred tortoise. Li Shou, minister of Huang Ti, the Yellow Emperor, about 2597 B.C., wrote his famous work on the "Nine Sections 九章." Lui Hui in his Commentary on the "Nine Sections" places the birth of algebra at this time, since the eighth of these sections contained linear equations of two or more unknowns and positive and negative numbers. The Chinese were acquainted with quadratic equations in the second and first centuries B.C., solved simultaneous equations in the third century, and equations of the third degree in the beginning of the T'ang Dynasty. Their method of solving quadratics and cubic equations sprang from their old process of extracting square and cube roots.

The unknown quantity being represented by the element *t'ien* (heaven), the solution of an equation was termed the *t'ien yuen* 天元, the " Celestial Element Method." We cannot tell when it arose. Chu, in his *Suan-hsüeh Ch'i-mêng* 算學啓蒙 or "Introduction to Mathematical Studies," published in 1299, solves problems by this method but his work contains nothing in advance of his predecessors. The *Suan-hsüeh* was merely a primer and evidently written as a text-book.

" The Precious Mirror " was a decided advance over the *Suan-hsüeh*. It was known during the Ming Dynasty but not understood, so little mention was made of it by other writers. It the " Precious Mirror " Chu extends the elements to four as indicated, thus carrying algebra to the loftiest height it was destined to reach. In simultaneous equations containing all the elements the different expressions were set up on different calculating boards and the elements were gradually eliminated by addition and subtraction. Another marked feature of his work, and probably the most remarkable, is that his solution of higher degree equations was identical with that of Horner's Method of Root Extension, published in Europe in 1819, over five hundred years later.

The " Precious Mirror " contains Elimination of Elements in Simultaneous Equations, Multiplication and Division by the Synthetic Method, and the Summation of Series. The arithmetical triangle, known in mathematics as Pascal's triangle for determining the co-efficients of the terms of higher degree equations, and published by Pascal in 1643, but engraved on the title page of Apinius August 9, 1526, and, according to Cantor, known to the Arabs in the eleventh century, appears in this work. Here Chu calls it " the ancient method of determining the co-efficients of higher degree equations."

The " Precious Mirror " is made up of three books corresponding to the three talents ; four elements corresponding to the four seasons ; and twenty-four sections, corresponding to the festivals, as the equinoxes and solstices. The whole book contains two hundred and eighty-eight problems.

I have frequently been asked the nature of the problems. Book I contains eighteen problems on right triangles, eighteen on plane figures, nine on piece goods, six on store houses for grain, and thirteen on equations with fractional roots.

Book II contains one hundred and three problems ; two " mixed as you please," nine containing the square and circle, fourteen on making circles with the three values of π, twenty on areas, eighteen on surveying with right triangles, twelve miscellaneous problems in poetic form, seven on different shaped piles of hay, seven on bundles of arrows, nine on land measurements, five on " men summoned according to need." (That is for building public works, or the army).

Book III contains twenty problems on different shaped piles of fruit, nineteen on figures within figures, eight on simultaneous equations with positive and negative roots, thirteen miscellaneous problems, twelve containing expressions in two unknown quantities, twenty-one on "left and right—you meet elements," eleven containing expressions in three unknown quantities, and six containing expressions in four unknowns.

You will note that the word expression is used and not equation. The absolute term of the expression is called *t'ai shu*, indicated by the character *t'ai*, an abbreviation of the term *t'ai-chi*, the great origin, which is the beginning of all elements. In an expression of higher degree than the second, the last term is called the *yü*, modified by the degree of the expression, that is, cubic *yü*, if it is a cubic expression, or the fourth power of *yü* if it is an expression of the fourth degree. The first term is always called *shih*, and the second *fang*, the terms between the second and the last are called *lien*, and are distinguished by their order; that is, first *lien*, second *lien*, and so on. The first *lien* is called the upper *lien* and the last *lien* the lower *lien*.

Take this problem for illustration :—

Problem.—The volume of an observatory is 18,528 *ch'ih*. It is said that the square root of the sum of the lengths of the upper and lower bases is less than the width of the upper base by thirteen *ch'ih*, equals the difference between the upper and lower lengths, and is one-third of the length ; the difference between the widths of the upper and lower bases is six *ch'ih*. It is required to enlarge the observatory into a circular form, using the diagonals as the diameters of the bases, and to complete the work in one day. The rate of one man's work is twenty-seven *ch'ih*. How many men shall be employed for the work ? Find also the lengths of the bases and the height and the volume of the arcs.

Process.—Let the element *t'ien* be the width of the upper base of the observatory. From the statement we have 18,774 for the negative *shih*, 702 for the negative *fang*, 391 for the positive first *lien*, 36 for the negative last *lien* and 1 for the positive *yü*, an expression of the fourth degree whose root is the width of the upper base. The other dimensions required can be obtained from this width. The expression in modern form is the equation

$$18{,}774 - 702X + 391X^2 - 36X^3 + X^4 = 0.$$

The problems in poetic form remind us of the famous Indian problems of Brahmagupta.

"I heard some one talking in the midst of the fog,
Saying he had a mixture of wine, weak and strong,
That it took three of the weak to intoxicate one,
While three could be intoxicated by means of the strong.
Fifty persons were found lying on their seats after twelve *tou* were gone.
May I ask those skilled in calculation,
Those who come from the four directions,
How much was in the mixture, the weak and the strong ?"

The work accomplished in algebra by the Chinese is another witness to the fact that they early developed a high stage of culture and that largely without extraneous influences.

While we recognize the great achievements of Modern Mathematics, we must not underestimate the great work accomplished by the pioneers. We marvel that the Greeks were able to achieve what they did in geometry with their clumsy system of notation; we marvel that the Chinese were able to reach the height they did with calculating board and rods. Chinese algebra is no less a product of the Chinese genius than Greek geometry is of the Greek.